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STRESS DISTRIBUTION IN TRIMODAL SAMPLES

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Abstract. The distribution of stress between coarser and finer particles in gap-graded soils is considered a key factor contributing to the risk of internal instability or suffusion, amongst other soil properties. In reality soils can have more complex size distributions than being purely bimodal. In this study, the discrete element method was used to investigate the stress distribution of trimodal gap-graded materials with different grading curves. The quantification of stresses and contacts forces at particle scale data indicates that the stress distribution in trimodal materials are influenced by the percentage of fines, the proportion of the medium fraction, and the initial density. Specifically, when the stress transfer within trimodal material was partitioned into six contacts classes, the results indicate that the stress carried out by each contact type is strongly associated with their percentage fractions and the size ratio between the different particle types.

1 INTRODUCTION

The particle size distribution (PSD), i.e. the cumulative distribution by particle mass (volume) of particle sizes (diameters), is one of the most basic ways to characterize a soil in both geotechnical research and practice. Soils are described as uniformly graded, broadly graded or gap-graded depending on the shape of the particle size distribution. While the PSD shape is understood to influence the engineering behaviour of soil, the fundamental mechanics of the influence of PSD shape on behaviour are poorly understood.

Restricting consideration to non-plastic, cohesionless soils, it is clear that there have been a large number of experimental studies looking at gap-graded soils. In most cases a finer grained sand was mixed with a coarser grained sand or fines were added to a host sand so that the resulting mixtures in both cases were almost bimodal. The proportion of the overall mass taken up by the finer grains (typically representing silts and clays) is then termed the fines content

(FC). Zuo and Baudet (2015) give a review of the relevant literature, focusing on the idea of a transitional FC, where the material behaviour transitions from being dominated by the coarse grains to become dominated by the finer grains. Developing on ideas put forward by Skempton and Brogan (1994), Shire et al. (2014) carried out a series of DEM simulations on gap-graded samples of spheres. They calculated the average stress in the finer grains normalized by the overall applied stress, this approximates to the proportion of stress transmitted by the finer grains. Shire's data showed that the extent to which the fines participate in stress transmission depends on the fines content and on the ratio between the coarse and finer grain sizes. The data do not support the concept of a single transitional FC, rather a more gradual transition between coarse- and fines-dominated behaviour.

Natural geological deposits of purely bimodal material are rare and so a comprehensive and relevant understanding of stress distribution in materials needs to consider more complex particle size distributions. More robust analyses of the effect of PSD shape on the mechanical behaviour of soil mixtures may therefore require varying PSDs in a more systematic way, from uniformly graded to well graded PSDs. In a first step to develop this broader perspective, this study considers trimodal materials, with fine, coarse and medium-sized grains. Rather than considering the stresses in the particles, the sample is partitioned into six classes: stress transmitted via (i) coarse-coarse particle contacts, (ii) coarse-medium contacts, (iii) medium-medium contacts, (iv) medium-fine contacts, (v) coarse-fine contacts and (vi) fine-fine contacts. DEM samples are compressed from an initial non-contacting cloud of grains to an isotropic stress of 500 kPa. Initially a purely bimodal material containing 25% fines is considered and the proportion of the medium-sized grains is then systematically increased.

2 PARTICLE SIZE DISTRIBUTION CONSIDERED

A total of 9 trimodal specimens were created; their grading curves are shown in Fig. 1. The minimum and maximum particle diameters used in each simulation specimen were 0.076 mm and 0.425 mm, respectively. Three size ratios (i.e. the size ratio between coarse and fines-sized grains, SR_{cf}; the size ratio between Coarse and medium-sized grains, SR_{cm}; the size ratio between medium and fines-sized grains, SR_{mf}) and three particles fraction (i.e. fines fraction; medium fraction; coarse fraction) are considered in this study. As shown in Fig.1, for all 9 grading curves, the size ratio SR_{cf} is constant, which is 5.6 in this study. For clarity, each of those specimens is named as A Tri B%_C%, where A represents a size ratio SR_{mf}, B% is a number representing a fines fraction percent by mass/volume, and C% represents a medium fraction. For instance, '1.54 Tri 25.0%_50.0%' indicates a trimodal specimen with the fines fraction of 25% and the medium fraction of 50%, and the size ratio SR_{mf} of this specimen is 1.54.

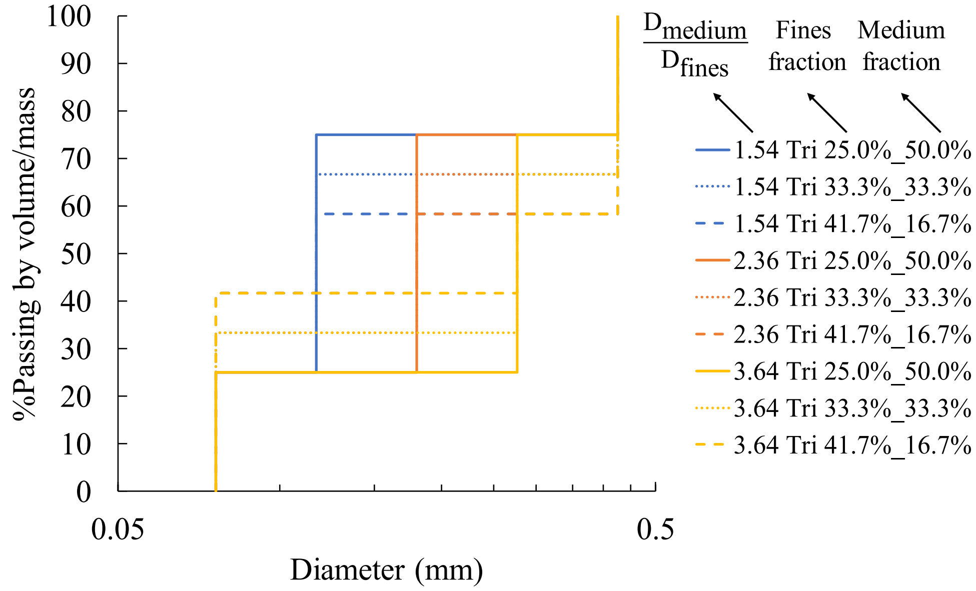

Figure 1: Particle size distribution of simulation specimens

3 DEM SIMULATIONS AND SETUP

All DEM simulations were conducted on cubic samples by means of a modified version of the open-source DEM code Granular LAMMPS (Plimpton, 1995). Periodic boundary conditions were employed to generate all specimens due to its efficiency in reducing boundary effects (e.g. Shire et al., 2014). A simplified Hertz-Mindlin contact model was used, the basic input simulation parameters were shear modulus (G = 29.17 GPa), particle density (2670 kg/m^3), Poisson's ratio ($\nu = 0.2$). These input parameters were used in previous DEM works (e.g. Huang et al., 2014) and were similar to experimentally derived values (e.g. Barreto, 2008). All DEM specimens were initially created in random positions by means of an in-house placement code (Fig. 2), followed by periodic isotropic compression through the method proposed by Cundall (1988). A stress-controlled algorithm was used to achieve the target isotropic mean effective stress of 500 kPa. To investigate the density effect, three inter-particle friction coefficient μ were used for isotropic compression: (1) $\mu = 0.001$, referring to a dense condition; (2) $\mu = 0.1$, referring to a moderate condition; (3) $\mu = 0.3$. referring to a loose condition.

All simulations were carried out by means of the high-performance computing (HPC) service CX1 at Imperial College London. Although particle shape is known to be a significant factor in influencing soil behaviour, the high computational cost of simulations necessitated the selection of spherical particles. All simulations were terminated when the mean effective stress reached the desired values of 500 kPa, with coordination number, void ratio remaining constant.

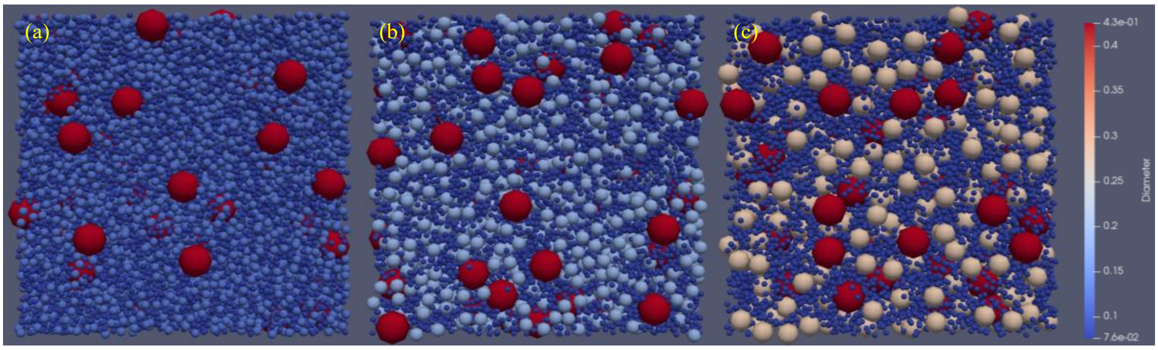

Figure 2: Typical generated simulation specimens: (a) 1.54 Tri 25.0%_50.0%; (b) 2.36 Tri 25.0%_50%; (c)3.64 Tri 25.0%_50%

4 RESULTS AND DISCUSSION

4.1 Detailed data for 3.64 Tri 25.0%_50.0%

A simulation test of 3.64 Tri 25.0%_50.0% (particle size distribution illustrated on Fig. 3a) with $\mu = 0.1$ (i.e. a moderately dense condition) is considered in detail to better understand the stress transfer within trimodal materials. Fig. 3b shows the cumulative distribution of particle connectivity by number for simulation, where the connectivity is the number of contacts involving an individual particle. The distributions for the coarse, medium and fines-sized grains are presented separately. As shown in Fig. 3b, the connectivity values for both the coarse and medium-sized grains are always greater than 0, which suggests that all of the particles in these size fractions are active in stress transmission when subjected to isotropic compression. However, for approximately 50% of the fines-size grains, the connectivity number is 0, which suggests those particles sit loosely in the void space and do not transfer stress. Fig. 3c shows the cumulative distribution of particle mean stresses by number for the simulation. Fig. 3c confirms that the coarse and medium-sized grains transmit stress, while approximately 50% of the fines-sized grains are not active in transferring stress.

Fig. 3d presents the cumulative distribution of contact forces by number for this simulation. As illustrated in Fig 4a, for trimodal materials, a total of six contacts classes support the whole structure: (i) coarse-coarse contacts (C-C contacts), (ii) coarse-medium contacts (C-M contacts), (iii) medium-medium contacts (M-M contacts), (iv) medium-fines contacts (M-F contacts), (v) coarse-fine contacts (C-F contacts) and (vi) fines-fines contacts (F-F contacts). As shown in Fig. 3d, for approximately 99% of the F-F contacts, M-F contacts and C-F contacts, the contact forces are lower than 0.04 N, on the contrary, the contact force values of C-C contacts, C-M contacts and M-M contacts have a much broader distribution. Those results show that those contact forces that do exist involving the fine-sized grains are very small compared to other grains, i.e. even where the fine grains have contacts, their contribution to the overall stress transmission is very small.

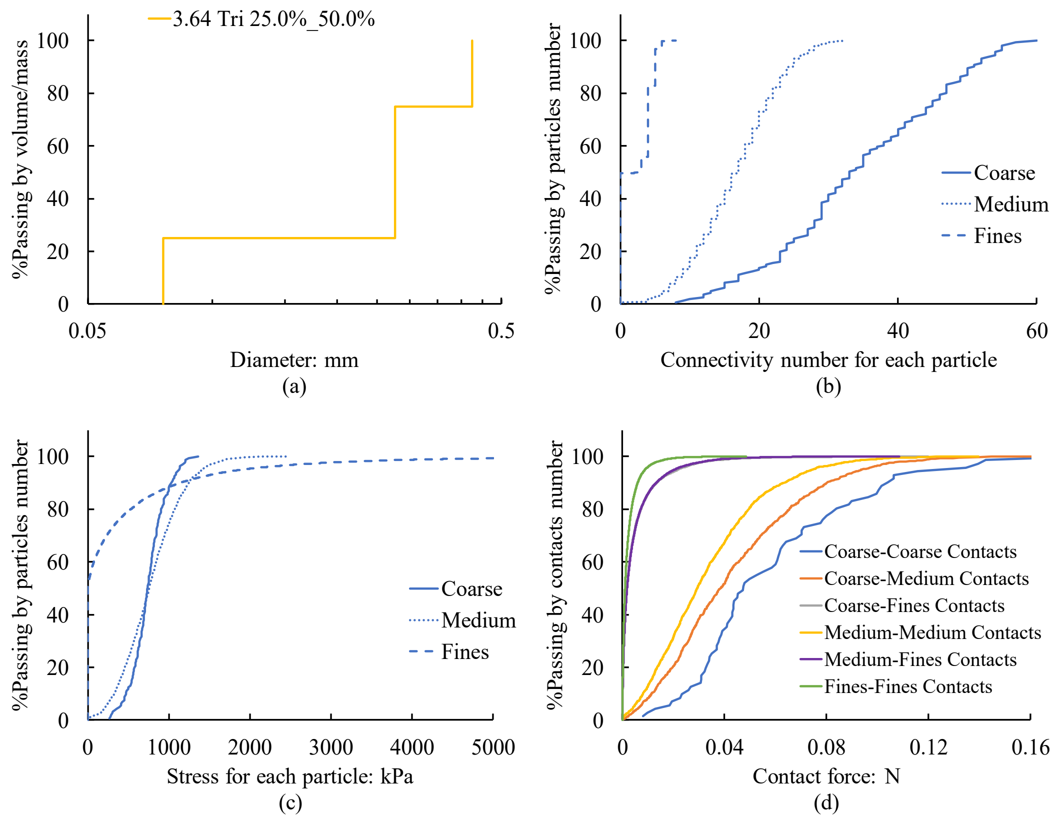

Figure 3: Detailed data for 3.64 Tri 25%_50% test: (a) particle size distribution; (b) connectivity number distribution; (c) stress level distribution; (d) contact force distribution

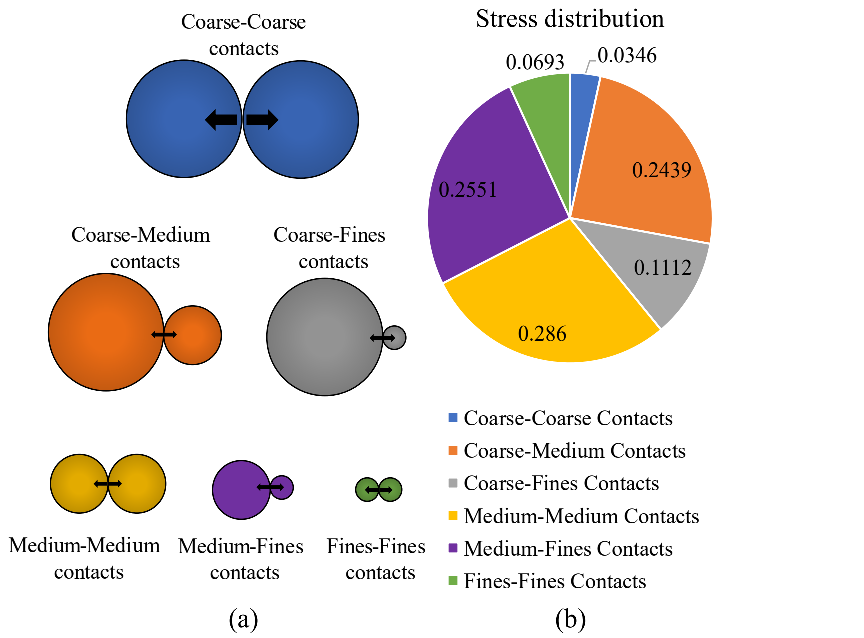

Figure 4: Analysis of contacts for 3.64 Tri 25%_50% test: (a) six contacts types; (b) stress distribution determined by contact forces

4.2 Analysis of contacts and stress distribution

In DEM, the contacts force data can be used to calculate the stresses within the material. As illustrated in O'Sullivan (2011) (for example) the 3D stress tensor is calculated by equation (1):

$$\sigma_{ij} = \frac{1}{V}\sum_{C=1}^{N_{c,V}} f_i^c l_j^c \qquad (1)$$

Where N_c,V is the total number of contacs in the volume V, f_i^c =(f_x^c f_y^c f_z^c) is the force vector for contact c and l_j^c =(l_x^c l_y^c l_z^c) is the branch vector for contact c. The contribution of each set of contacts to the overall stress tensor can be determined by restricting the summation to contacts within that set (i.e. for the C-C contacts the summation considers only C-C contact forces). Fig. 4b illustrates the proportion of the mean stress ($\frac{1}{3}\sigma_{ii}$) carried by each set of contacts for the simulation test above (i.e. 3.64 Tri 25.0%_50.0%, μ = 0.1). The medium fraction accounts for largest proportion of the specimen by mass and so it can be seen that C-M contacts, M-M contacts and M-F contacts contribute to the most proportion of the stress transmission of the structure. Specifically, the M-M contacts transmit the largest proportion (i.e. 0.286) of the stresses, which is 0.286 × 500 kPa = 143 kPa.

On the contrary, the stresses carried out by C-C contacts are relatively small compared to those of other contacts, which may be attributed to that both medium and fines-sized grains separate the coarse-sized grains during isotropic compression. Furthermore, F-F contacts account for a small proportion (i.e. 0.0693) of the stress transmission of the structure, which agrees with results identified in Fig. 3b and Fig. 3c.

4.3 Density effect

Shire et al. (2014) carried out a series of DEM simulations on bimodal gap-graded soils and highlighted the significant influence of initial density on the stress transmission, therefore, it is worthwhile to investigate the density effect for trimodal materials. As shown in Fig. 5, the initial density strongly affects stress transmission in the trimodal material considered, for instance, Fig. 5a shows that the connectivity number of approximately 14% particles is 0 when the specimen is in the dense condition. On the contrary, in the loose condition, approximately 83% particles do not connect with other particles, which suggests those particles are not active in support the structure. The similar tendency is also identified in Fig. 5b and Fig. 5c. Prior analyses of bi-modal materials suggest that the susceptibility of the stress distribution to changes in density will depend on the proportion of each particle type and the ratios of particle sizes.

Fig. 5d illustrates the variation in the stress distribution by contact type with density. AS the density increases from the loose state, it can be seen that the stresses carried out by M-F and F-F contacts significantly increase, while the stresses transferred by C-C and C-M contacts decrease at the same time. Those results highlight the significant influence of initial density on the stress distribution for within the trimodal material considered here.

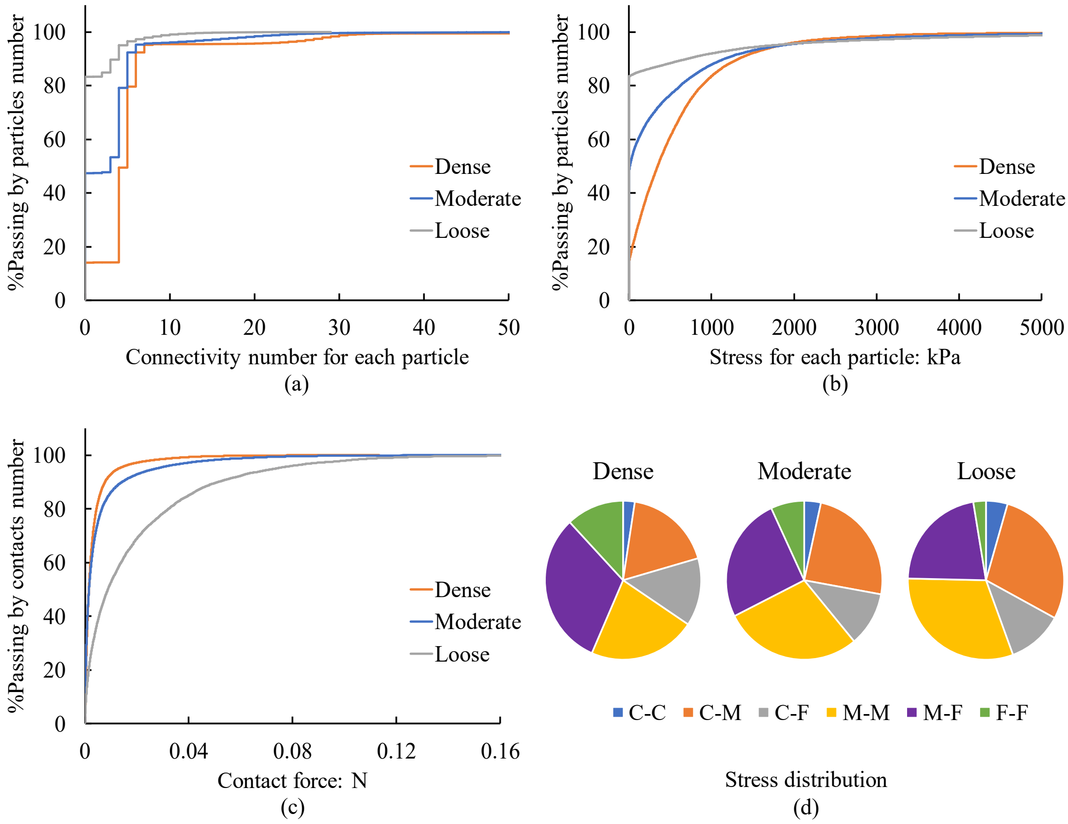

Figure 5: 3.64 Tri 25%_50% tests with three density conditions: (a) connectivity number distribution; (b) stress level distribution; (c) contact force distribution; (d) stress distribution determined by contact forces

4.4 Fines and medium content effects

Various combinations of fine and medium fractions considered to better understand how the proportion of the different fractions influences the overall stress transmission. Typical simulation tests with $\mu = 0.3$ were selected as illustrated in Fig. 6a, the size ratio SR_{mf} of three specimens is fixed of 3.64 with the fines and medium fractions changing in a wide range. Fig. 6b shows the stress distributions for three specimens, the results indicate that the stresses carried out by C-F and F-F contacts significantly increases with an increase in the fines fraction, while the amount of the overall stresses carried out by the C-M and M-M contacts declines as the proportion of the medium fraction decreases. Furthermore, the stresses transferred by M-F contacts vary significantly.

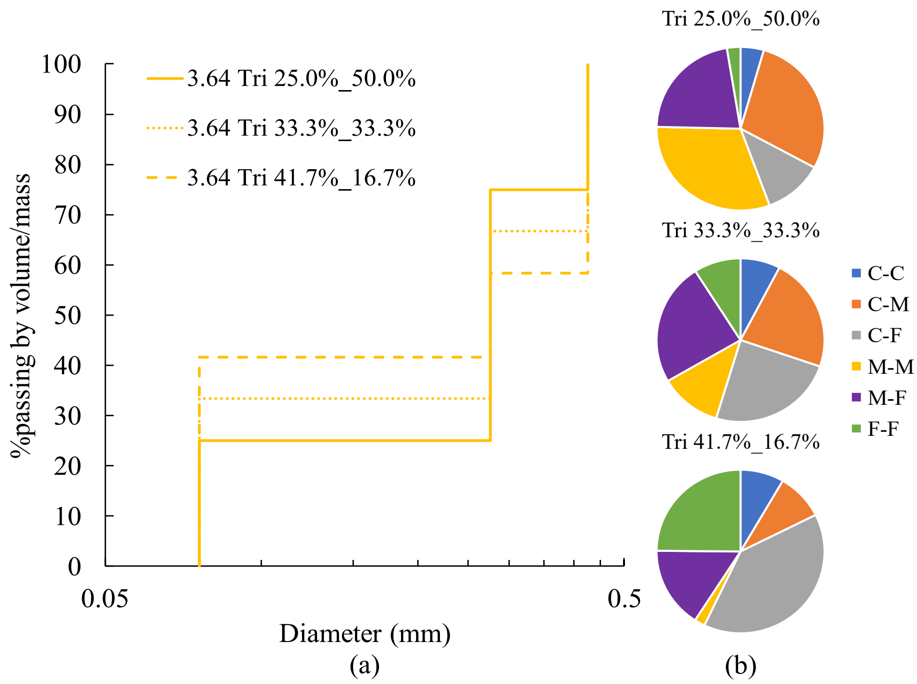

Figure 6: Fines and medium content effects: (a) particle size distributions; (b) stress distribution

4.5 Size ratio effect

The size ratio effect was studied by keeping the fine and medium fractions fixed, as illustrated in Fig. 7a. Restricting consideration to a loose condition, Fig. 7b considers the stress distributions three specimens with size ratios SR_{mf} changing from 1.54 to 3.64. The results show that the stress transmission is strongly associated with the size ratio SR_{mf}. For instance, the stresses carried out by M-F contacts significantly decrease with the rise of the size ratio SR_{mf}, which suggests that the interaction between medium and fines-sized grains decreases with the size ratio SR_{mf} increase.

On the contrary, the stresses transferred by C-M contacts increase with increasing size ratio SR_{cm}. Furthermore, it can be seen that the stresses transmitted by the C-F contacts fluctuated in a narrow range for those three tests, which may be attributed to the constant size ratio SR_{cf} for these specimens.

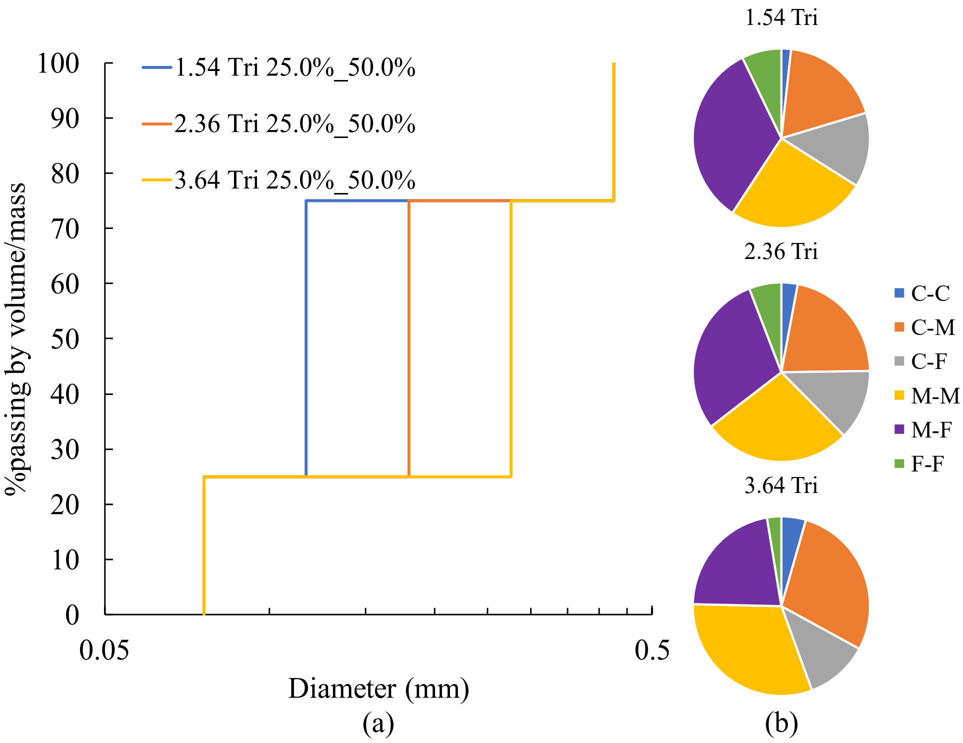

Figure 7: Size ratio effect: (a) particle size distributions; (b) stress distribution

5 CONCLUSIONS

This paper presents a total of 27 DEM simulation at three initial densities to study the stress distribution on trimodal materials of isotropic compression. The results highlighted the significance influence of initial density on the stress distribution of trimodal materials. In addition, the results show that the stress distributions in trimodal materials are strongly associated with fines fraction, medium fraction and the size ratios among those grains.

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